Two Particle Ghost Interference Demystified

Pravabati Chingangbam*
Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad-211019, India.

Tabish Qureshi†
Department of Physics, Jamia Millia Islamia, New Delhi-110025, India.

The two-photon ghost interference experiment, generalized to the case of massive particles, is theoretically analyzed. It is argued that the experiment is intimately connected to a double-slit interference experiment where, the which-way information exists. The reason for not observing first order interference behind the double-slit, is clarified. It is shown that the underlying mechanism for the appearance of ghost interference is, the more familiar, quantum erasure.

PACS numbers: 03.65.Ud ; 03.65.Ta

I. INTRODUCTION

A puzzling experiment was reported by Strekalov et al [1] which was appropriately called ghost interference. The experiment was a dramatic demonstration of the nonlocal nature of correlations that exist in spatially separated, entangled particles.

We briefly describe Strekalov et al's experiment. A source S sends two entangled photons from SPDC, which we call photon 1 and photon 2 (see Fig. 1). A double-slit is placed in the path of photon 1. Surprisingly, no first order interference is observed for photon 1. For photon 2, first order interference is neither expected, nor is it seen. Then a lone detector is placed behind the double slit, and the position of photon 2 is detected *in coincidence with the detector behind the double slit*. Photon 2 shows an interference pattern which is very similar to a double-slit interference pattern, even though there is no double-slit in the path of photon 2. Another curious thing about the observed interference is that it is the same as what one would observe if one were to replace the lone detector behind the double slit, with the source of light, and the SPDC source were absent. In other words, the standard Young's double slit interference formula works, if the distance is taken to be the distance between the screen (detector) on which photon 2 registers, right through the SPDC source crystal, to the double slit. Photon 2 never passes through the region between the source S and the double slit. Zeilinger's group performed a ghost interference experiment using an optical grating [2]. Ghost interference has become a subject of experimental and theoretical research attention [3, 4].

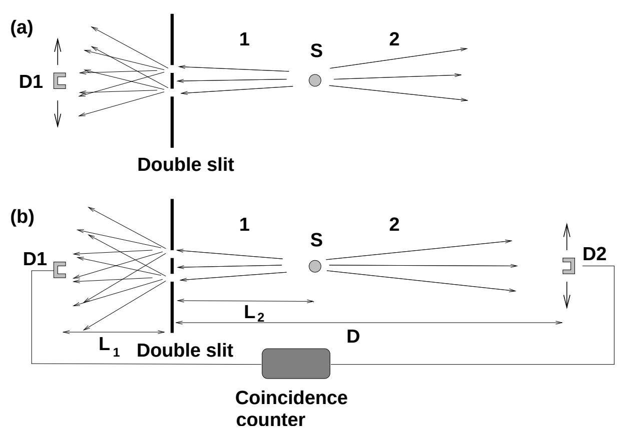

FIG. 1: An SPDC source generates photon pairs - one goes left, and the other right. (a) Putting a double slit in the path of photon 1 results in no interference. (b) Counting of photon 2 in coincidence with a *fixed* detector D1 clicking, results in a ghost interference.

Strekalov et al attribute the absence of first order interference in photon 1 to the large momentum spread of photon 1 - "the 'blurring out' of the first order interference is due to the considerably large angular propagation uncertainty of a single SPDC photon" [1, 3]. We will show that there is a more fundamental reason why a first order interference can never be observed in an experiment with entangled photons. For explaining ghost interference, Strekalov et al present a geometrical model which satisfactorily reproduces the observed pattern. However, we believe that the mechanism behind the emergence of ghost interference can be understood better by looking at it from a different perspective. In this paper, we present this new way of looking at ghost interference. We will also show that an interference for photon 1 can be observed, under certain conditions.

II. THEORETICAL ANALYSIS

At the heart of this effect is the phenomenon of entanglement, which applies as much to massive particles, as to photons. For clarity, we will analyze the ghost interference experiment using entangled particles, rather than photons. The results can easily be applied to the case of photons. Let there be two particles of identical mass,

*Electronic address: prava@hri.res.in
†Electronic address: tabish@jamia-physics.net

generated at the source S, in an entangled state. We assume the entangled state to be of the following form:

$$\Psi(y_1, y_2) = A\int_{-\infty}^{\infty} dp e^{-p^2/4\sigma^2} e^{-ipy_2/\hbar} e^{ipy_1/\hbar} e^{-\frac{(y_1+y_2)^2}{4\Omega^2}}, \tag{1}$$

where A is a constant necessary for the normalization of Ψ. Without the $e^{-(y_1+y_2)^2/4\Omega^2}$ term, the state (1) is infinitely extended in space. So, we introduce the term $e^{-(y_1+y_2)^2/4\Omega^2}$, for a realistic situation. This is a momentum entangled state, which is fairly general, barring the use of Gaussian functions. Integration over p can be performed to obtain:

$$\Psi(y_1, y_2) = \frac{1}{\sqrt{\pi\Omega\hbar/\sigma}} e^{-(y_1-y_2)^2\sigma^2/\hbar^2} e^{-(y_1+y_2)^2/4\Omega^2}. \tag{2}$$

The physical meaning of the constants σ and Ω will become clear if we calculate the uncertainty in position and momentum of the two particles. The uncertainty in momenta of the two particles is given by

$$\Delta p_{1y} = \Delta p_{2y} = \sqrt{\sigma^2 + \frac{\hbar^2}{4\Omega^2}}. \tag{3}$$

The position uncertainty of the two particles is given by

$$\Delta y_1 = \Delta y_2 = \sqrt{\Omega^2 + \hbar^2/4\sigma^2}. \tag{4}$$

So, now we know the position and momentum spread of both the particles in this state. With time, the particles travel along the positive and negative x-axis. The motion in the x-direction is disjoint from the evolution in the y-direction, and is unaffected by entanglement. So, in order to see the effect of the double slit on particle 1, we will assume that state evolves for a time t_0 before particle 1 reaches the double-slit.

A. Double-slit and which-way information

Imagine the slit to be a position filter - it allows portions of the wavefunction in front of the slit, to go through. Let us assume that what passes through a slit is a localized Gaussian packet, whose width is the width of the slit. So, if the two slits are A and B, the packets which pass through will be, say, $|\phi_A(y_1)\rangle$ and $|\phi_B(y_1)\rangle$, respectively.

The entangled state at time t_0, $|\Psi(y_1, y_2, t_0)\rangle$, can then be expanded in terms of components parallel to $|\phi_A(y_1)\rangle$ and $|\phi_B(y_1)\rangle$, and perpendicular to those. We can write

$$|\Psi(y_1, y_2, t_0)\rangle = |\phi_A\rangle\langle\phi_A|\Psi\rangle + |\phi_B\rangle\langle\phi_B|\Psi\rangle + |\chi\rangle\langle\chi|\Psi\rangle. \tag{5}$$

where $|\chi(y_1)\rangle$ represents rest of the states in the Hilbert space, orthogonal to $|\phi_A(y_1)\rangle$ and $|\phi_B(y_1)\rangle$. So, the states of particle 2 that one has to calculate are

$$\begin{aligned}
\psi_A(y_2) &= \langle\phi_A(y_1)|\Psi(y_1, y_2, t_0)\rangle \\
\psi_B(y_2) &= \langle\phi_B(y_1)|\Psi(y_1, y_2, t_0)\rangle \\
\psi_\chi(y_2) &= \langle\chi(y_1)|\Psi(y_1, y_2, t_0)\rangle
\end{aligned} \tag{6}$$

So, the state we get after particle 1 crosses the double-slit is:

$$|\Psi(y_1, y_2)\rangle = |\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle + |\chi\rangle|\Psi_\chi\rangle, \tag{7}$$

where $|\phi_A\rangle$ and $|\phi_B\rangle$ are states of particle 1, and $|\psi_A\rangle$ and $|\psi_B\rangle$ are states of particle 2. The first two terms represent the amplitudes of particle 1 passing through the slits, and the last term represents it getting reflected/blocked. Because of the linearity of Schrodinger equation, these two pieces of the wavefunction will evolve independently, without affecting each other. Because we are interested only in the particle 1, which passes through the slit, we might as well throw away the term which represents particle 1 not passing through the slits. If we do that, we have to normalize the remaining part of the wavefunction, which looks like

$$|\Psi(y_1, y_2)\rangle = \frac{1}{C}(|\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle), \tag{8}$$

where $C = \sqrt{\langle\psi_A|\psi_A\rangle + \langle\psi_B|\psi_B\rangle}$. After coming out of the slits, particle 1 travels in time, and reaches the D1 where it is detected. In the mean time, particle 2 also undergoes a unitary time evolution. The state at a later time t, can be written as

$$\Psi(y_1, y_2, t) = \frac{1}{C}[\phi_A(y_1, t)\hat{U}_2(t)|\psi_A\rangle + \phi_B(y_1, t)\hat{U}_2(t)|\psi_B\rangle], \tag{9}$$

where $\hat{U}_2(t)$ represents the time evolution operator for particle 2. The state (9) represents particle 1 passing through a double-slit, but if $|\psi_A\rangle$ and $|\psi_B\rangle$ are orthogonal, the amplitudes of particle 1 passing through the two slits are correlated with two distinguishable states of particle 2. Hence, in principle, a measurement on particle 2 can reveal which slit particle 1 passed through. According to the Complementarity principle, no interference can be observed in such a situation. So, no interference can be seen in particle 1 because particle 2 carries the "which-way" information about particle 1. This is the fundamental reason for photon 1 not showing interference in the ghost interference experiment, and not its large momentum spread.

The argument presented above can be cast in a mathematical language by calculating the probability density of particle 1 on the screen, which is given by:

$$\begin{aligned}
P(y_1) &= \int_{-\infty}^{\infty} \Psi^*(y_1, y_2, t)\Psi(y_1, y_2, t) dy_2 \\
&= \frac{1}{|C|^2}\Big(|\phi_A(y_1, t)|^2\langle\psi_A|\hat{U}_2^\dagger(t)\hat{U}_2(t)|\psi_A\rangle \\
&\quad + |\phi_B(y_1, t)|^2\langle\psi_B|\hat{U}_2^\dagger(t)\hat{U}_2(t)|\psi_B\rangle \\
&\quad + \phi_A^*(y_1, t)\phi_B(y_1, t)\langle\psi_A|\hat{U}_2^\dagger(t)\hat{U}_2(t)|\psi_B\rangle \\
&\quad + \phi_B^*(y_1, t)\phi_A(y_1, t)\langle\psi_B|\hat{U}_2^\dagger(t)\hat{U}_2(t)|\psi_A\rangle\Big)
\end{aligned}$$

$$= \frac{1}{|C|^2}\left(|\phi_A(y_1,t)|^2\langle\psi_A|\psi_A\rangle + |\phi_B(y_1,t)|^2\langle\psi_B|\psi_B\rangle \right.$$
$$+\phi_A^*(y_1,t)\phi_B(y_1,t)\langle\psi_A|\psi_B\rangle$$
$$\left. +\phi_B^*(y_1,t)\phi_A(y_1,t)\langle\psi_B|\psi_A\rangle\right) \quad (10)$$

If $|\psi_A\rangle$ and $|\psi_B\rangle$ are orthogonal, the last two terms in the above equation, which represent interference, will disappear. Hence no first order interference can be seen for particle 1.

B. Entanglement and virtual double-slit

In order to explain ghost interference, we need to consider explicit functional form of $|\phi_A\rangle$, $|\phi_B\rangle$. In the following, we assume that $|\phi_A\rangle$, $|\phi_B\rangle$, are Gaussian functions in space:

$$\phi_A(y_1) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}} e^{-(y_1-y_0)^2/\epsilon^2}$$
$$\phi_B(y_1) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}} e^{-(y_1+y_0)^2/\epsilon^2}, \quad (11)$$

where $\pm y_0$ is the y-position of slit A and B, respectively, and ϵ their widths. Thus, the distance between the two slits is $2y_0$. To account for the full phase information in the particles, one should also consider the evolution for a time t_0, before particle 1 reaches the double-slit. The state of the entangled system, after this time evolution, can be calculated using the Hamiltonian governing the time evolution, given by

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y_2^2} \quad (12)$$

After a time t_0, (2) assumes the form

$$\Psi(y_1,y_2,t_0) = \frac{1}{\sqrt{\frac{\pi}{2}(\Omega + \frac{i\hbar t_0}{m\Omega})(\hbar/\sigma + \frac{4i\hbar t_0}{m\hbar/\sigma})}} \times$$
$$\exp\left[\frac{-(y_1-y_2)^2}{\hbar^2/\sigma^2 + \frac{4i\hbar t_0}{m}}\right]\exp\left[\frac{-(y_1+y_2)^2}{(\Omega^2 + \frac{i\hbar t_0}{m})}\right] \quad (13)$$

We wish to point out that the use of (12) is not an absolute necessity for obtaining the time evolution of the state. For example, if one considers the particle to be an envelope of waves , the time evolution can be obtained easily. In that case, $\left(\frac{d^2\omega(k)}{dk^2}\right)_{k_0}$, where k_0 is the wave-vector value where $\omega(k)$ peaks, plays the role of $\hbar/m$. The time evolution for a photon state can be obtained similarly [5]. Using (6) and (13), wavefunctions $|\psi_A\rangle$, $|\psi_B\rangle$ can be calculated, which, after normalization, have the form

$$\psi_A(y_2) = C_2 e^{-\frac{(y_2-y_0')^2}{\Gamma^2}}$$
$$\psi_B(y_2) = C_2 e^{-\frac{(y_2+y_0')^2}{\Gamma^2}}, \quad (14)$$

where $C_2 = \frac{1}{(\pi/2)^{1/4}\sqrt{\Gamma}}$,

$$y_0' = \frac{y_0}{\frac{4\Omega^2\sigma^2/\hbar^2+1}{4\Omega^2\sigma^2/\hbar^2-1} + \frac{4\epsilon^2}{4\Omega^2-\hbar^2/\sigma^2}}, \quad (15)$$

and

$$\Gamma^2 = \frac{\frac{\hbar^2}{\sigma^2}(1+\frac{\epsilon^2+2i\hbar t_0/m}{4\Omega^2}) + \epsilon^2 + 2i\hbar t_0/m}{1 + \frac{\epsilon^2+2i\hbar t_0/m}{\Omega^2} + \frac{\hbar^2}{4\Omega^2\sigma^2}} + \frac{2i\hbar t_0}{m}. \quad (16)$$

The state which passes through the double slit, now assumes the form

$$\Psi_r(y_1,y_2) = C_1 e^{-(y_1-y_0)^2/\epsilon^2} C_2 e^{-\frac{(y_2-y_0')^2}{\Gamma^2}}$$
$$+C_1 e^{-(y_1+y_0)^2/\epsilon^2} C_2 e^{-\frac{(y_2+y_0')^2}{\Gamma^2}} \quad (17)$$

Equation (17) represents two wave-packets of particle 1, of width ϵ, and localized at $\pm y_0$, entangled with two wave-packets of particle 2, of width $\frac{\sqrt{2}|\Gamma|^2}{\sqrt{\Gamma^2+\Gamma^{*2}}}$, localized at $\pm y_0'$. So, because of entanglement, particle 2 also behaves as if it has passed through a double-slit of separation $2y_0'$. In other words, because of entanglement, particle 1 passing through the double-slit, creates a *virtual double-slit* for particle 2. This view also agrees with the observed optical imaging by means of entangled photons [6]. It appears natural that particle 2, passing through this virtual double-slit, should show an interference pattern. However, this can happen only when the wave-packets overlap, after evolving in time.

Before reaching detector D2, particle 2 evolves for a time t. The time evolution, governed by (12), transforms the state (17) to

$$\Psi_r(y_1,y_2,t) = C_1(t)e^{-\frac{(y_1-y_0)^2}{\epsilon^2+2i\hbar t/m}} C_2(t)e^{-\frac{(y_2-y_0')^2}{\Gamma^2+2i\hbar t/m}}$$
$$+C_1(t)e^{-\frac{(y_1+y_0)^2}{\epsilon^2+2i\hbar t/m}} C_2(t)e^{-\frac{(y_2+y_0')^2}{\Gamma^2+2i\hbar t/m}}, \quad (18)$$

where

$$C_1(t) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon + 2i\hbar t/m\epsilon}},$$
$$C_2(t) = \frac{1}{(\pi/2)^{1/4}\sqrt{\Gamma + 2i\hbar t/m\Gamma}}. \quad (19)$$

Before proceeding further, we need to simplify the expression for Γ. We assume the spatial extent of the wave-function $\Psi(y_1,y_2)$ to be large, namely, $\Omega \gg \epsilon$ and $\Omega \gg \hbar/\sigma$. In this limit,

$$\Gamma^2 \approx \gamma^2 + 4i\hbar t_0/m, \quad (20)$$

where $\gamma^2 = \epsilon^2 + \hbar^2/\sigma^2$ and $y_0' \approx y_0$. We are now in a position to calculate the probability of finding particle 1 at y_1 and particle 2 at y_2. This is given by

$$|\Psi_r(y_1,y_2,t)|^2 = |C_1(t)C_2(t)|^2 \times ($$

$$
\begin{aligned}
& e^{-\frac{2(y_1-y_0)^2}{\epsilon^2+(2\hbar t/m\epsilon)^2}} e^{-\frac{2(y_2-y_0')^2}{\gamma^2+(2\hbar(t+2t_0)/m\gamma)^2}} \\
& +e^{-\frac{2(y_1+y_0)^2}{\epsilon^2+(2\hbar t/m\epsilon)^2}} e^{-\frac{2(y_2+y_0')^2}{\gamma^2+(2\hbar(t+2t_0)/m\gamma)^2}} \\
& +e^{-\frac{2(y_1^2+y_0^2)}{\epsilon^2+(2\hbar t/m\epsilon)^2}-\frac{2(y_2^2+y_0'^2)}{\gamma^2+(2\hbar(t+2t_0)/m\gamma)^2}} \\
& \times 2\cos\left[\theta_1 y_1+\theta_2 y_2\right]), \qquad (21)
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_1 &= \frac{8y_0\hbar t/m}{\epsilon^4+4\hbar^2t^2/m^2}, \\
\theta_2 &= \frac{8y_0\hbar(t+2t_0)/m}{\gamma^4+4\hbar^2(t+2t_0)^2/m^2}. \qquad (22)
\end{aligned}
$$

We can now make contact with the ghost interference experiment, where detector D1 is kept fixed and detector D2 is scanned along the y-axis. If we fix y_1, the cosine term in (21) represents oscillations as a function of y_2. This implies that if particle 2 is detected in coincidence with particle 1 being detected at a fixed position y_1, then it shows interference. This is ghost interference. In the expression for θ_2, γ is a measure of the width of the virtual slits created because of particle 1 passing through the double-slit. So, it is clear that the ghost interference is an effect due to the virtual slits formed for particle 2, because of a measurement on the spatially separated particle 1. This is what Karl Popper, the philosopher of science, was uncomfortable about. He had proposed an experiment aimed at disproving such a nonlocal influence [7]. But ghost interference shows much more than what Popper's experiment aimed to disprove.

C. Erasing the which-way information

Coming back to ghost interference, one might wonder that if the virtual slits are created by particle 1 passing through the double-slit, what is the need for fixing detector D1 and doing a coincident count. The answer is, particle 1 carries which-way information about particle 2. Meaning, particle 1 can be potentially detected in such a manner that would tell us which virtual slit, A or B, did particle 2 pass through. And Bohr's complementarity principle tells us that in such a situation, interference cannot be observed. By detecting particle 1 at a fixed position, where contributions from both the slits are present, we add the two contributions, and thus *erase* the information about which slit particle 1, and particle 2, passed through. Once the which-way information is erased, the interference can come back, and it does. So, the mechanism behind the appearance of ghost interference is essentially *quantum erasure* [8].

Scully Englert and Walther proposed a setup for quantum eraser, where the which-way detector is a two-state system [9]. Quantum erasure is performed when the particles are detected in coincidence with one of the two states of the which-way detector. Corresponding to the two states of the which-way detector, two interference patterns are obtained which are complementary, meaning, they add up to give no interference. In the case of ghost interference, the role of the which-way detector states is played by the position of D1, which is a continuous variable. From (21), one can see that for a fixed y_1, the term $\theta_1 y_1$ acts as an extra phase for the cosine function in y_2. Thus, the whole interference pattern is shifted, depending on the position of D1 (see Fig. 2), so that when all the D1 positions are added, it results in the destruction of the interference pattern. This is the reason why, in Strekalov et al's experiment, no interference for photon 2 is observed without coincident counting with a fixed D1.

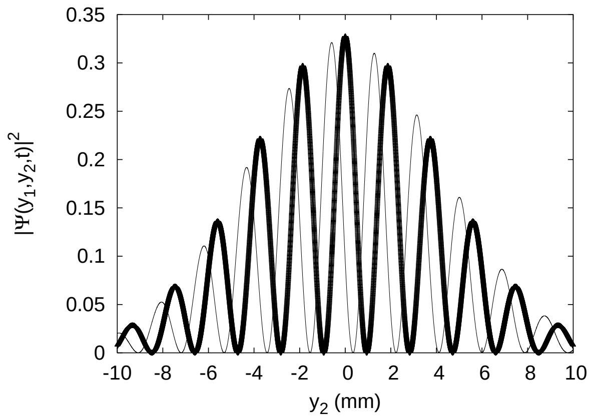

FIG. 2: Probability density of particle 2 as a function of the position of detector D2, for $\lambda_d = 314$ nm, $D = 3$ m, $L_1 = 1$ m, $2y_0 = 0.5$ mm and $\epsilon = 0.05$ mm. The dark pattern corresponds to $y_1 = 0$ mm, and the lighter pattern corresponds to $y_1 = 0.2$ mm.

D. Where is the virtual slit located?

One must have already noticed something strange about (21), namely, in the terms for particle 2, the time which appears is $t + 2t_0$ as opposed to just t for particle 1. In the actual experiment, time is not what is the meaningful quantity - it is the distance the particle travels, that is relevant. Let us translate our results to the situation where one just measures the distance. For that we assume that particle 2 travels along the x-axis, with a momentum p. In time t_0, both particle 1 and particle 2 travel a distance L_2. During time t, particle 1 travels a distance L_1 to reach D1, and particle 2 travels the same distance to reach D2. So, the time $t + 2t_0$ corresponds to the distance between the double-slit and D2, that is, D. Using this strategy, we can write $\hbar(t + 2t_0)/m = \hbar v(t + 2t_0)/p = \lambda_d v(t + 2t_0)/2\pi = \lambda_d D/2\pi$, where λ_d is the d'Broglie wavelength of the particle and v its velocity. The expression $\lambda_d D/2\pi$ will also hold for a photon provided, one uses the wavelength of the photon in place of λ_d. The probability of coincident click of D1 and D2

is given by

$$
\begin{aligned}
P(y_1,y_2) \;=\; & |\Psi_r(y_1,y_2,t)|^2 = |C_1(t)C_2(t)|^2 \times (\\
& \exp\left[-\frac{2(y_1-y_0)^2}{\epsilon^2+(\lambda_d L_1/\pi\epsilon)^2} - \frac{2(y_2-y_0)^2}{\gamma^2+(\lambda_d D/\pi\gamma)^2}\right] \\
+ \; & \exp\left[-\frac{2(y_1+y_0)^2}{\epsilon^2+(\lambda_d L_1/m\epsilon)^2} - \frac{2(y_2+y_0)^2}{\gamma^2+(\lambda_d D/\pi\gamma)^2}\right] \\
+ \; & \exp\left[-\frac{2(y_1^2+y_0^2)}{\epsilon^2+(\lambda_d L_1/\pi\epsilon)^2} - \frac{2(y_2^2+y_0^2)}{\gamma^2+(\lambda_d D/\pi\gamma)^2}\right] \\
& \times 2\cos\left[\theta_1 y_1 + \theta_2 y_2\right]), \qquad (23)
\end{aligned}
$$

where $\theta_1 = \frac{4y_0\lambda_d L_1/\pi}{\epsilon^4+\lambda_d^2 L_1^2/\pi^2}$, $\theta_2 = \frac{4y_0\lambda_d D/\pi}{\gamma^4+\lambda_d^2 D^2/\pi^2}$. Equation (23) tells us that the fringe width of the pattern for particle 2 is given by

$$
w_2 = \frac{2\pi}{\theta_2} = 2\pi\frac{\lambda_d^2 D^2/4\pi^2+\gamma^4/4}{2y_0\lambda_d D/2\pi} = \frac{\lambda_d D}{2y_0} + \frac{\gamma^4\pi}{4y_0\lambda_d D} \qquad (24)
$$

For $\gamma^2 \ll \lambda_d D$, we get the familiar Young's double-slit interference formula,

$$
w_2 \approx \frac{\lambda_d D}{2y_0}, \qquad (25)
$$

where $2y_0$ is the separation between the slits. Notice that D is the strange distance from the detector D2, right through the source, to the double slit. Particle 2 never passes through the region between the source and the double-slit. This is exactly what was observed in Strekalov et al's experiment. Although the virtual double-slit for particle 2 comes into being only after particle 2 travels a distance L_2 from the source, the particle carries with itself the phase information of its evolution from the source for the time t_0. Because of coincident counting, the change in phase because of the evolution of particle 1 is added to that of particle 2, and *it appears as if* particle 2 traveled a distance $2L_2$, which is double the actually traveled distance. So, we see that although the virtual double-slit comes into being after particle 1 enters the real double-slit, for all practical purposes, *it appears as if* the virtual double-slit is located exactly at the real double-slit, *behind the source*. We should also mention that for values of various parameters corresponding to Strekalov et al's experiment, their results are faithfully reproduced by (23).

E. Interference for particle 1

Eqn. (23) is reasonably symmetric in y_1 and y_2, except for the difference in the widths of the real and virtual slits, and L_1 appearing for particle 1, and D appearing for particle 2. It is but natural to expect that by fixing D2 at some y_2, and counting particle 1 in coincidence with D2, should show an interference. The fringe width of the interference pattern is given by

$$
w_1 = \frac{2\pi}{\theta_1} = 2\pi\frac{\lambda_d^2 L_1^2/4\pi^2+\epsilon^4/4}{2y_0\lambda_d L_1/2\pi} \approx \frac{\lambda_d L_1}{2y_0} \qquad (26)
$$

The fringe width is exactly what one would expect from a conventional first order interference. In this sense, this pattern is not as spectacular as that for particle 2. The term $\theta_2 y_2$ now acts as an additional phase of the cosine, and this leads to a shift in the interference pattern for particle 1 if the position of D2 is changed.

III. CONCLUSION

From the preceding analysis, we conclude that in the ghost interference experiment, the reason for the absence of first order interference for particle 1 is that the which-way information for particle 1, is carried by particle 2. By complementarity, no interference can be observed in such a situation, in principle. Particle 2 can show interference because it experiences a virtual double-slit due to particle 1 passing through the double-slit. However, particle 1 carries which-way information about particle 2, and that washes out any potential interference. By fixing D1 and doing a coincident count of particle 2, one is erasing the which-way information. This quantum erasure leads to the appearance of ghost interference in particle 2. A corollary of the result is that particle 1 can also show interference if it is detected in coincidence with a fixed D2. The general analysis presented here shows that ghost interference can be observed for entangled massive particles too.

[1] D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, *Phys. Rev. Lett.* **74**, 3600-3603 (1995).
[2] R. Christanell, W. Weinfurter, and A. Zeilinger, *The Technical Digest of the European Quantum Electronic Conference, EQEC'93*, Florence, 1993 (unpublished), p. 872.
[3] M. D'Angelo, Y-H. Kim, S. P. Kulik and Y. Shih, *Phys. Rev. Lett.* **92**, 233601 (2004).
[4] S. Thanvanthri and M. H. Rubin, *Phys. Rev. A* **70**, 063811 (2004).
[5] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge, University Press, 1995).
[6] T.B. Pittman, Y.H. Shih, D.V. Strekalov, A.V. Sergienko, *Phys. Rev. A*, **52**, R3429 (1995).
[7] "Realism in quantum mechanics and a new version of the EPR experiment", K. R. Popper in *Open Questions in Quantum Physics*, edited by G. Tarozzi and A. van der Merwe (Dordrecht, Reidel, 1985).
[8] M. O. Scully and K. Drühl, *Phys. Rev. A* **25**, 2208 (1982).
[9] M. O. Scully, B.-G. Englert and H. Walther, *Nature (London)* **351**, 111 (1991).